

Relic neutrino asymmetry, CMB and large-scale structure

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We consider some consequences of the presence of a cosmological lepton asymmetry in the form of neutrinos. A relic neutrino degeneracy enhances the contribution of massive neutrinos to the present energy density of the Universe, and modifies the power spectrum of radiation and matter. Comparing with current observations of cosmic microwave background anisotropies and large scale structure, we derive some constraints on the relic neutrino degeneracy and on the spectral index in the case of a flat Universe with a cosmological constant.

1. Introduction

It is generally assumed that our Universe contains an approximately equal amount of leptons and antileptons. The lepton asymmetry would be of the same order as the baryon asymmetry, which is very small as required by Big Bang Nucleosynthesis (BBN) considerations. The existence of a large lepton asymmetry is restricted to be in the form of neutrinos from the requirement of universal electric neutrality, and the possibility of a large neutrino asymmetry is still open. From a particle physics point of view, a lepton asymmetry can be generated by an Affleck-Dine mechanism [1] without producing a large baryon asymmetry (see ref. [2] for a recent model), or even by active-sterile neutrino oscillations after the electroweak phase transition [3].

We have studied some cosmological implications of relic degenerate neutrinos [4] (here degenerate refers to neutrino-antineutrino asymmetry, not to mass degeneracy). We do not consider any specific model for generating such an asymmetry, and just assume that it was created well before neutrinos decouple from the rest of the plasma. An asymmetry of order one or larger can have crucial effects on the global evolution of the Universe. Among other effects, it changes the decoupling temperature of neutrinos, the primordial production of light elements at BBN, the time

of equality between radiation and matter, or the contribution of relic neutrinos to the present energy density of the Universe. The latter changes affect the evolution of perturbations in the Universe. We focus on the anisotropies of the Cosmic Microwave Background (CMB), and on the distribution of Large Scale Structure (LSS). We calculate the power spectrum of both quantities, in the case of massless degenerate neutrinos, and also for neutrinos with a mass of 0.07 eV, as suggested to explain the experimental evidence of atmospheric neutrino oscillations at Super-Kamiokande [5].

The effect of neutrino degeneracy on the LSS power spectrum was studied in ref. [6], as a way of improving the agreement with observations of mixed dark matter models with eV neutrinos, in the case of high values of the Hubble parameter. Adams & Sarkar [7] calculated the CMB anisotropies and the matter power spectrum, and compared them with observations in the $\Omega_\Lambda = 0$ case for massless degenerate neutrinos. More recently, Kinney & Riotto [8] also calculated the CMB anisotropies for massless degenerate neutrinos in the $\Omega_\Lambda = 0.7$ case.

2. Energy density of massive degenerate neutrinos

The energy density of one species of massive degenerate neutrinos and antineutrinos, described by the distribution functions f_ν and $f_{\bar{\nu}}$, is (we use $\hbar = c = k_B = 1$ units)

$$\rho_\nu + \rho_{\bar{\nu}} = \int_0^\infty \frac{dp}{2\pi^2} p^2 \sqrt{p^2 + m_\nu^2} (f_\nu(p) + f_{\bar{\nu}}(p)) \quad (1)$$

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valid at any moment. Here p is the magnitude of the 3-momentum and m_ν is the neutrino mass.

When the early Universe was hot enough, the neutrinos were in equilibrium with the rest of the plasma via the weak interactions. In that case the distribution functions f_ν and $f_{\bar{\nu}}$ changed with the Universe expansion, keeping the form of a Fermi-Dirac distribution,

$$f_{\nu,\bar{\nu}}(p) = \frac{1}{\exp\left(\frac{p}{T_\nu} \mp \frac{\mu}{T_\nu}\right) + 1} \quad (2)$$

Here μ is the neutrino chemical potential, which is nonzero if a neutrino-antineutrino asymmetry has been previously produced. Later the neutrinos decoupled when they were still relativistic, and from that moment the neutrino momenta just changed according to the cosmological red-shift. If a is the expansion factor, the neutrino momentum decreases keeping ap constant. At the same time the neutrino degeneracy parameter $\xi \equiv \mu/T_\nu$ is conserved, with a value equal to that at the moment of decoupling. Therefore one can still calculate the energy density of neutrinos now from eq. (1) and eq. (2), replacing μ/T_ν by ξ and p/T_ν by $p/(y_\nu T_0)$, where $T_0 \simeq 2.726$ K and y_ν is the present ratio of neutrino and photon temperatures, which is not unity because once decoupled the neutrinos did not share the entropy transfer to photons from the successive particle annihilations that occurred in the early Universe.

In the presence of a significant neutrino degeneracy ξ the decoupling temperature $T(\xi)$ is higher than in the standard case, [9,10]. The reaction rate Γ of the weak processes, that keep the neutrinos in equilibrium with the other species, is reduced because some of the initial or final neutrino states will be occupied. The authors of ref. [10] found that the neutrino decoupling temperature is $T_{dec}(\xi) \approx 0.2\xi^{2/3} \exp(\xi/3)$ MeV (for ν_μ or ν_τ). Therefore if ξ is large enough, the degenerate neutrinos decouple before the temperature of the Universe drops below the different mass thresholds, and are not heated by the particle-antiparticle annihilations, reducing the ratio of neutrino and photon temperatures with respect to the standard value $y_\nu = (4/11)^{1/3}$.

The present contribution of these degenerate neutrinos to the energy density of the Universe can be parametrized as $\rho_\nu = 10^4 h^2 \Omega_\nu$ eV cm⁻³, where Ω_ν is the neutrino energy density in units of the critical density $\rho_c = 3H^2 M_P^2 / 8\pi$, $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass and $H = 100h$ Km s⁻¹ Mpc⁻¹ is the Hubble parameter. The value of ρ_ν can be calculated as a function of the neutrino mass and the neutrino degeneracy ξ , or equivalently the present neutrino asymmetry L_ν defined as the following ratio of number densities

$$L_\nu \equiv \frac{n_\nu - n_{\bar{\nu}}}{n_\gamma} = \frac{1}{12\zeta(3)} y_\nu^3 [\xi^3 + \pi^2 \xi] \quad (3)$$

We show³ in figure 1 the contours in the (m_ν, L_ν) plane that correspond to some particular values of $h^2 \Omega_\nu$. In the limit of small degeneracy (vertical lines) one recovers the well-known bound on the neutrino mass $m_\nu \lesssim 46$ eV for $h^2 \Omega_\nu = 0.5$. On the other hand, for very light neutrinos the horizontal lines set a maximum value on the neutrino degeneracy, that would correspond to a present neutrino chemical potential $\mu_0 \lesssim 7.4 \times 10^{-3}$ eV, also for $h^2 \Omega_\nu = 0.5$. In the intermediate region of the figures the neutrino energy density is $\rho_\nu \simeq m_\nu n_\nu(\xi)$ and the contours follow roughly the relation $L_\nu(m_\nu/\text{eV}) \simeq 24.2 h^2 \Omega_\nu$.

A similar calculation has been recently performed in reference [11]. Note however that the ratio of neutrino and photon temperatures was not properly taken into account for large ξ .

The presence of a neutrino degeneracy can modify the outcome of BBN (for a review see [12]). First a larger neutrino energy density increases the expansion rate of the Universe, thus enhancing the primordial abundance of ⁴He. This is valid for a nonzero ξ of any neutrino flavor. In addition if the degenerate neutrinos are of electron type, they have a direct influence over the weak processes that interconvert neutrons and protons. This last effect depends on the sign of ξ_{ν_e} , and one gets $-0.06 \lesssim \xi_{\nu_e} \lesssim 1.1$ [10], while a sufficiently long matter dominated epoch requires $|\xi_{\nu_\mu, \nu_\tau}| \lesssim 6.9$ [10]. This estimate agrees with our analysis in section 4 and places a limit shown by

³Here we assume $\xi > 0$, but the results are also valid for $\xi < 0$ provided that ξ and L_ν are understood as moduli.

the horizontal line in figure 1 in the case of degenerate ν_μ or ν_τ .

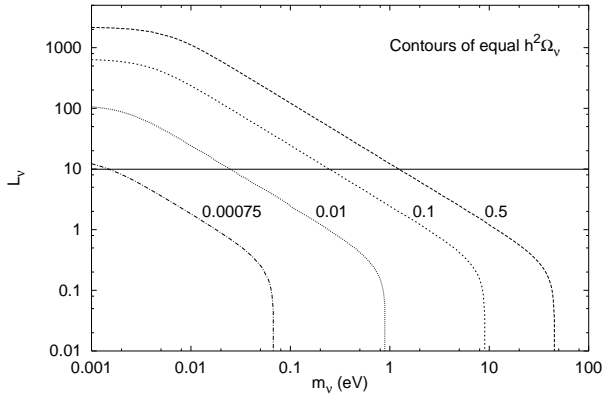


Figure 1. Present energy density of massive degenerate neutrinos as a function of the neutrino asymmetry.

3. Effects on the power spectra

We compute the power spectra of CMB anisotropies and LSS using the Boltzmann code `cmbfast` by Seljak & Zaldarriaga [13], adapted to the case of one family of degenerate neutrinos (ν , $\bar{\nu}$), with mass m_ν and degeneracy parameter ξ . Our modifications to the code are reviewed and explained in [4].

The effect of ξ and m_ν on the CMB anisotropy spectrum can be seen in figure 2. We choose a set of cosmological parameters ($h = 0.65$, $\Omega_b = 0.05$, $\Omega_\Lambda = 0.70$, $\Omega_{CDM} = 1 - \Omega_b - \Omega_\nu - \Omega_\Lambda$, $Q_{rms-ps} = 18 \mu\text{K}$, flat primordial spectrum, no reionization, no tensor contribution), and we vary ξ from 0 to 5, both in the case of massless degenerate neutrinos and degenerate neutrinos with $m_\nu = 0.07 \text{ eV}$.

Let us first comment the massless case. The main effect of ξ is to boost the amplitude of the first peak⁴. Indeed, increasing the energy density of radiation delays matter-radiation equality,

⁴In fact, this is not true for very large values of ξ , where recombination can take place still at the end of radiation domination, and anisotropies are suppressed. However in such a case the location of the first peak is $l \gtrsim 450$, and the matter power spectrum is strongly suppressed.

which is known to boost the acoustic peaks, and to shift them to higher multipoles, by a factor $((1 + a_{eq}/a_*)^{1/2} - (a_{eq}/a_*)^{1/2})^{-1}$ (a_{eq} increases with ξ , while the recombination scale factor a_* is almost independent of the radiation energy density). Secondary peaks are then more affected by diffusion damping at large l , and their amplitude can decrease with ξ .

In the case of degenerate neutrinos with $m_\nu = 0.07 \text{ eV}$, the results are quite similar in first approximation. Indeed, the effects described previously depend on the energy density of neutrinos at equality. At that time, they are still relativistic, and identical to massless neutrinos with equal degeneracy parameter. However, with a large degeneracy, Ω_ν today becomes significant: for $\xi = 5$, one has $\Omega_\nu = 0.028$, i.e. the same order of magnitude as Ω_b . Since we are studying flat models, Ω_ν must be compensated by less baryons, cold dark matter (CDM) or Ω_Λ . In our example, Ω_b and Ω_Λ are fixed, while Ω_{CDM} slightly decreases. This explains the small enhancement of the first peak compared to the massless case (3.4% for $\xi = 5$). Even if this effect is indirect, it is nevertheless detectable in principle, possibly by future satellite missions *MAP* and *Planck* (even if one does not impose the flatness condition, the effect of Ω_ν will be visible through a modification of the curvature).

We also plot in figure 2 the power spectrum $P(k)$, normalized on large scales to COBE. The effect of both parameters ξ and m_ν is now to suppress the power on small scales. Indeed, increasing ξ postpones matter-radiation equality, allowing less growth for fluctuations crossing the Hubble radius during radiation domination. Adding a small mass affects the recent evolution of fluctuations, and has now a direct effect: when the degenerate neutrinos become non-relativistic, their free-streaming suppresses the growth of fluctuations for scales within the Hubble radius. This effect, already known for non-degenerate neutrinos [14], is enhanced in the presence of a neutrino degeneracy, since the average neutrino momentum is shifted to larger values.

Our results for massless degenerate neutrinos can be compared with those of previous works. We found the same effect of ξ on the CMB for

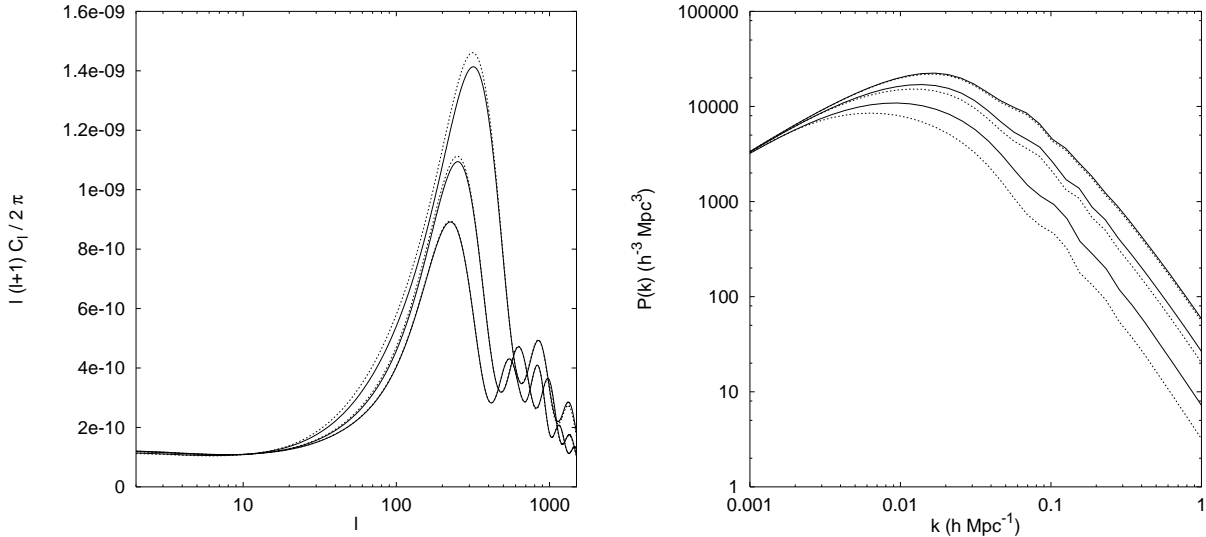


Figure 2. CMB anisotropy and matter power spectra for different models with one family of massless (solid lines) and $m_\nu = 0.07$ eV (dashed lines) degenerate neutrinos. From bottom to top (from top to bottom for $P(k)$), $\xi = 0, 3, 5$. Cosmological parameters are fixed as described in the text.

$\Omega_\Lambda = 0$ as in [7], while the revised results in [8] also agree with our calculations for $\Omega_\Lambda = 0.7$.

4. Comparison with observations

Since the degeneracy increases dramatically the amplitude of the first CMB peak, we expect large ξ values to be favored in the case of cosmological models known to predict systematically a low peak (unless a large blue tilt is invoked, which puts severe constraints on inflation). Our goal here is not to explore systematically all possibilities, but to briefly illustrate how ξ can be constrained by current observations for flat models with different values of Ω_Λ . Recent results from supernovae, combined with CMB constraints, favor flat models with $\Omega_\Lambda \sim 0.6 - 0.7$.

We choose a model with $h = 0.65$, $\Omega_b = 0.05$, $Q_{rms-ps} = 18 \mu\text{K}$, no reionization and no tensor contribution, and look for the allowed window in the space of free parameters (Ω_Λ, ξ, n) . The allowed window is defined as the intersection of regions preferred at the 95% confidence level by four independent experimental tests, based on σ_8 estimation, Stromlo-APM redshift survey, bulk ve-

locity reconstruction, and CMB anisotropy measurements. Details concerning these tests can be found in [4].

We plot in figure 3 the LSS and CMB allowed regions in (ξ, n) parameter space, for $\Omega_\Lambda = 0$ and 0.6. In the case of degenerate neutrinos with $m_\nu = 0.07$ eV, the LSS regions are slightly shifted at large ξ , since, as we saw, the effect of ξ is enhanced (dotted lines on the figure). The CMB regions do not show this distinction, given the smallness of the effect and the imprecision of the data. One can immediately see that LSS and CMB constraints on n are shifted in opposite direction with ξ : indeed, the effects of ξ and n both produce a higher CMB peak, while to a certain extent they compensate each other in $P(k)$. So, for $\Omega_\Lambda \geq 0.7$, a case in which a power spectrum normalized to both COBE and σ_8 yields a too high peak⁵, a neutrino degeneracy can only make things worse, and we find no allowed window at all. In the other extreme case $\Omega_\Lambda = 0$, it is well known that the amplitude required by σ_8 and the

⁵At least, for the values of the other cosmological parameters considered here. This situation can be easily improved, for instance, with $h = 0.7$.

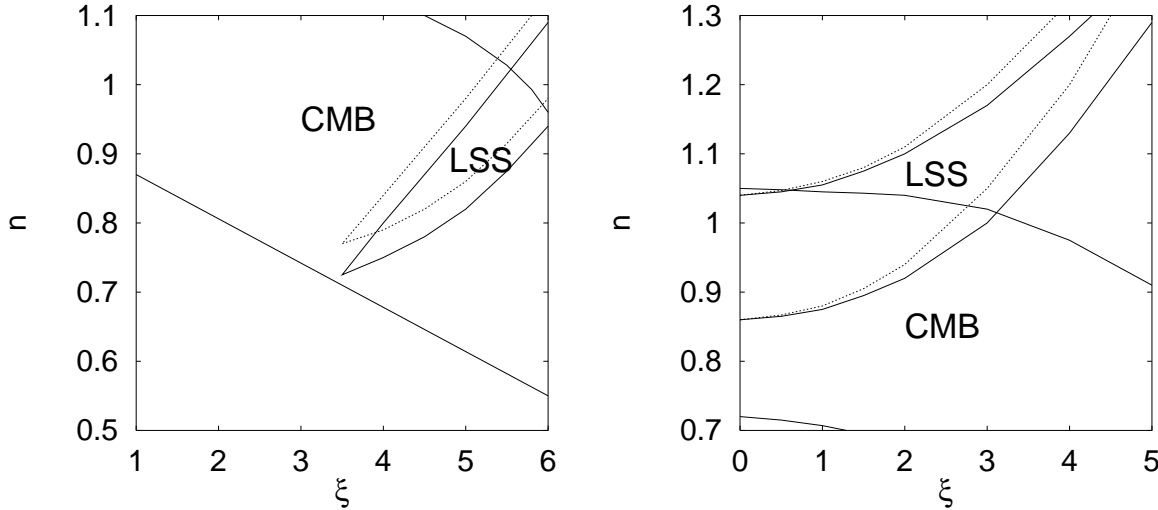


Figure 3. LSS and CMB constraints in (ξ, n) space for $\Omega_\Lambda = 0$ (left) and $\Omega_\Lambda = 0.6$ (right). The underlying cosmological model is flat, with $h = 0.65$, $\Omega_b = 0.05$, $Q_{rms-ps} = 18 \mu\text{K}$, no reionization, no tensor contribution. The allowed regions are those where the labels are. For LSS constraints, we can distinguish between degenerate neutrinos with $m_\nu = 0$ (solid lines) and $m_\nu = 0.07 \text{ eV}$ (dotted lines).

shape probed by redshift surveys favor different values of n . We find that the neutrino degeneracy can solve this problem with $\xi \gtrsim 3.5$, but the allowed window is cut at $\xi \simeq 6$ by CMB data, and we are left with an interesting region in which $\Omega_0 = 1$ models are viable. This result is consistent with [7]. However, current evidences for a low Ω_0 Universe are independent of the constraints used here, so there are not many motivations at the moment to consider this window seriously. Finally, for $\Omega_\Lambda = 0.5 - 0.6$, a good agreement is found up to $\xi \simeq 3$. This upper bound could marginally explain the generation of ultra-high energy cosmic rays by the annihilation of high-energetic neutrinos on relic neutrinos with mass $m_\nu = 0.07 \text{ eV}$ [15].

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